COMP/I-T/III/CBGS/Appl. Maths-III | 25-11-2016

Q.P. Code : 540701

(3 Hours) **[Total Marks : 80]**

NB: 1) Question No. 1 is compulsory.
2) Attempt any three of the remaining.
3) Figures to the right indicate full marks.

1. a) Find the Laplace transform of $te^{3t}\sin 4t$. **05**

b) Find half-range cosine series for $f(x)=e^x$, $0<x<1$. **05**

c) Is $f(z)=\frac{z}{\bar{z}}$ analytic? **05**

d) Prove that $\nabla x(\bar{a}x\nabla \log r) = 2\frac{(\bar{a}.\bar{r})\bar{r}}{r^4}$, where $\bar{a}$ is a constant vector. **05**

2. a) Find the Z-transform of $\frac{1}{(z-5)^3}$ if $|z|<5$. **06**

b) If $V=3x^2y+6xy-y^3$, show that V is harmonic & find the corresponding analytic function. **06**

c) Obtain Fourier series for the function **08**

$$f(x)=\begin{cases}1+\frac{2x}{\pi}, -\pi\le x\le 0\\ 1-\frac{2x}{\pi}, 0\le x\le \pi\end{cases}$$

hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots$

3. a) Find $L^{-1}\left[\frac{(s+2)^2}{(s^2+4s+8)^2}\right]$ using convolution theorem. **06**

b) Show that the set of functions **06**

$1, \sin\left(\frac{\pi x}{L}\right), \cos\left(\frac{\pi x}{L}\right), \sin\left(\frac{2\pi x}{L}\right), \cos\left(\frac{2\pi x}{L}\right), \ldots\ldots\ldots$

Form an orthogonal set in (-L,L) and construct an orthonormal set.

c) Verify Green's theorem for $\int_C (e^{2x}-xy^2)\,dx+(ye^x+y^2)dy$ **08**

Where C is the closed curve bounded by $y^2=x$ & $x^2=y$.

4. a) Find Laplace transform of $f(x)=K\frac{t}{T}$ for $0<t<T$ & $f(t)=f(t+T)$. **06**

b) Show that the vector, $\bar{F}=(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$ is irrotational and hence, find ϕ such that $\bar{F}=\nabla\phi$. **06**

c) Find Fourier series for f(x) in $(0, 2\pi)$, **08**

$$f(x)=\begin{cases}x, & 0\le x\le \pi\\ 2\pi-x, & \pi\le x\le 2\pi\end{cases}$$

hence deduce that

$\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots\ldots\ldots$

5. a) Use Gauss's Divergence theorem to evaluate **06**

$\iint_S \bar{N}.\bar{F}\,ds$ where $\bar{F}=2xi+xyj+zk$ over the region bounded by the cylinder $x^2+y^2=4$, $z=0$, $z=6$.

b) Find inverse Z – transform of $f(x)=\frac{z}{(z-1)(z-2)}, |z|>2$ **06**

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c) (i) Find $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$ **08**

(ii) Find $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$

6. a) Solve $(D^2+3D+2)\,y = 2(t^2+t+1)$ with $y(0) = 2$ & $y'(0) = 0$. **06**

b) Find the bilinear transformation which maps the points 0, i, -2i of z-plane onto the points -4i, ∞, 0 respectively of w-plane. Also obtain fixed points of the transformation. **06**

c) Find Fourier sine integral of **08**

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

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Course: S.E. (SEM - III) (REV.-2012) (CBSGS) (COMPUTER ENGG.) COMMON WITH (INFORMATION TECHNOLOGY)(Prog T1123 CW 1723A)

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Correction:

Q. 2(a)

Read As:

Find the **inverse** Z-transform of.....

Instead of :

Find the Z-transform of.....

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Correction:

Q. 4(a)

Read As

Find laplace transform of f(t)=

Instead of:

Find laplace transform of f(x)=

Query Update time: 25/11/2016 04:31 PM